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Q/ Establish the equivalence of mass and Energy. Describle the circumference which led the Concept of mass and energy.

Ans Einstein's mass-Energy equivalance Relation: Let the mass m of the body moving with velocity v relative to stationary observer varies which v is ~~go~~ given as $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$

Here m_0 is rest mass of the body and c be the velocity of light. The variation of mass with velocity has modified the idea of energy so that a relationship can be ~~def~~ derived between mass and energy. The relationship can be derived directly from the definition of K.E T of a moving body

$$T = \int_0^x F\,dx$$

But $F = \frac{d}{dt}(mv)$

$$T = \int_0^x \frac{d}{dt}(mv)\,dx = \int_0^x d(mv)\frac{dx}{dt} \quad \because \frac{dx}{dt} = v$$

$$T = \int_0^v v\,d(mv)$$

Let $m = m_0/\sqrt{1-v^2/c^2}$

$$\therefore T = \int_0^v v\,d\left[\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right]$$

$$= \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} - m_0\int_0^v \frac{v\,dv}{\sqrt{1-v^2/c^2}}$$

$$= \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} - m_0\left[-c^2\sqrt{1-v^2/c^2}\right]_0^v$$

$$= \frac{m_o v^2}{\sqrt{1-v^2/c^2}} + m_o c^2 \left[\sqrt{1-v^2/c^2} \right]_0^v$$

$$= \frac{m_o v^2}{\sqrt{1-v^2/c^2}} + m_o c^2 \left[\sqrt{1-v^2/c^2} - 1 \right]$$

$$= \frac{m_o v^2}{\sqrt{1-v^2/c^2}} + m_o c^2 \left[\sqrt{1-v^2/c^2} \right] - m_o c^2$$

$$= \frac{m_o}{\sqrt{1-v^2/c^2}} \left[v^2 + c^2(1-v^2/c^2) \right. - m_o c^2$$

$$= \frac{m_o c^2}{\sqrt{1-v^2/c^2}} - m_o c^2 = mc^2 - m_o c^2 = (m-m_o)c^2$$

The K.E of a body is equal to the relativistic increase in mass of the body over the rest mass multiplied by the square the velocity of light. It means that a body possesses energy $m_o c^2$, the total energy E of the body is therefore the sum of the K.E and its rest energy

$$E = T + E_0 = (m-m_o)c^2 + m_o c^2 = mc^2$$

$$E = mc^2$$

It is the famous Einsteen's mass energy relation which shows that the amount of energy mc^2 is associated with mass m of a system or conversly, a system with T.E 'E' has associated with inertial mass E/c^2

Importance of the mass energy relation.

Since $m = \dfrac{m_o}{\sqrt{1-v^2/c^2}} = m_o \left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$

for values $\frac{v}{c}$

$$m = m_0\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right)$$ using Binomial Theorem

or $m = m_0 + \frac{1}{2}\frac{v^2}{c^2}m_0$ (approx)

from above relation it can be seen that the total ~~ma~~ inertial mass of a particle moving with velocity v relative to observer is the sum of

(I) its rest mass (m_0)

(II) an additional mass equal to $\frac{E}{c^2}$

So this is clear that the relation of energy to a system increases the inertial mass of the system ~~a~~ in other word mass may be appear as energy and energy as mass. The scope of the law of conservation of energy may be widened to into what may be called the laws of conservation of mass - energy. In any interaction it is neither mass alone nor energy alone that remains conserved but mass inclusive of energy in terms of mass or energy inclusive of mass in terms of the energy. That is totally conserved